SIRPY · Problem 8 (Stiff) — Where the Guarantee Ends, and How Far It Extends

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An ongoing series. We are building a public library of problems SIRPY has solved — each entry is one problem, the exact input we handed the solver, the exact output it returned, and an honest read of what happened. No slides. No hand-waving. Numbers you can check.

The problem

$$y'' + 101\,y' + 100\,y = 0, \qquad y(0) = 1, \qquad y'(0) = -1 \tag{1}$$

This is a classic **stiff** linear initial-value problem, and the stiffness is visible by hand. Assume $y = e^{rx}$ and substitute:

$$r^2 + 101r + 100 = 0 \implies (r+1)(r+100) = 0 \implies r = -1,\ r = -100.$$

The equation carries two modes at once: a **fast** mode e^{-100x} that dies almost instantly, and a **slow** mode e^{-x} that persists. With the initial data above, the exact solution is the slow mode alone:

$$y(x) = e^{-x}.$$

That is what makes stiffness treacherous. The fast mode is *absent from the answer* — but it is present in the equation, and any numerical error excites it. A method that does not respect the $r = -100$ timescale must either take absurdly small steps or watch small perturbations get amplified. The solution is trivial; the *equation* is hostile.

We entered it three ways, escalating the demand each time.

Part 1 — No upper bound at all

The first experiment: state the problem and ask for nothing else. No interval end, no tuning.

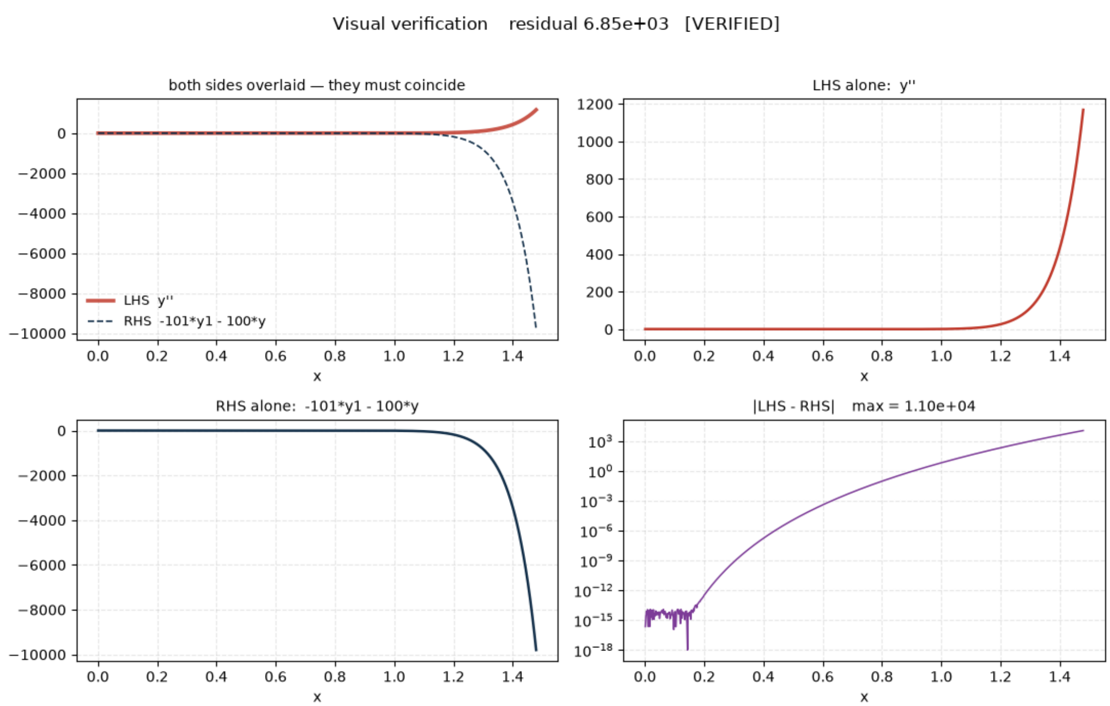
```
r = solve_ivp(order=2, f_str="-101*y1 - 100*y",
              x0=0.0, ic={0: 1, 1: -1},
              iterations=20)
```

What came back:

```
Estimated reliable range: |x - 0.0| < 1.644
VERIFIED on [0, 0.44398] to 1e-6 relative residual.
Values past 0.44398 are unaudited. For a longer range, pass x1=<target>.
```

Read the second and third lines together, because they are the point of this entry. SIRPY did not simply return a series and leave you to guess how far to trust it. It drew two boundaries: an estimated reliable range for the series, and a shorter interval on which the result is **verified** — and then it said, plainly, that everything beyond that is *unaudited*.

“Unaudited” is a word chosen with care. It does not mean wrong; it means *unchecked*, and a solver that marks the difference is one whose “VERIFIED” actually means something.



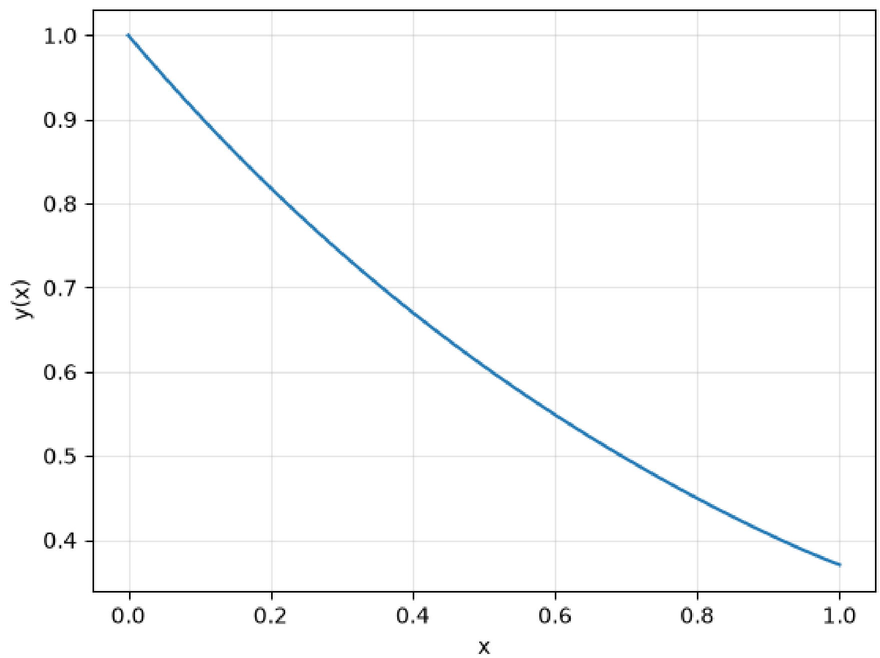


Figure 1: Solution from the open-ended run.

Part 2 — Extended to $x = 1$

Next, we asked for the interval $[0, 1]$ — beyond what a single series covered reliably:

```
r = solve_ivp(order=2, f_str="-101*y1 - 100*y",
              x0=0.0, x1=1, ic={0: 1, 1: -1},
              iterations=20)
```

```
INFO: the interval [0.0, 1] exceeds a single series' reliable range;
      the solution was continued analytically over 9 segments
      (piecewise power series - still fully analytic on every piece).
INFO: run summary - series degree 20, iterations=20, 9 series segment(s).
INFO: verification by substitution - VERIFIED: satisfies the equation
      to 3.74e-09 on [0, 1].
```

No manual segmentation and no mesh design. When one series could not cover the interval within its trustworthy radius, SIRPY continued the solution analytically across nine pieces — each an exact series, each sized by where the previous one stops being reliable — and then verified the assembled solution globally by substitution, to 3.74×10^{-9}.

The result is still an analytic object end to end: readable, differentiable, evaluable at any point. The segmentation is how honesty about a series' reliable range coexists with covering the interval you actually asked for.

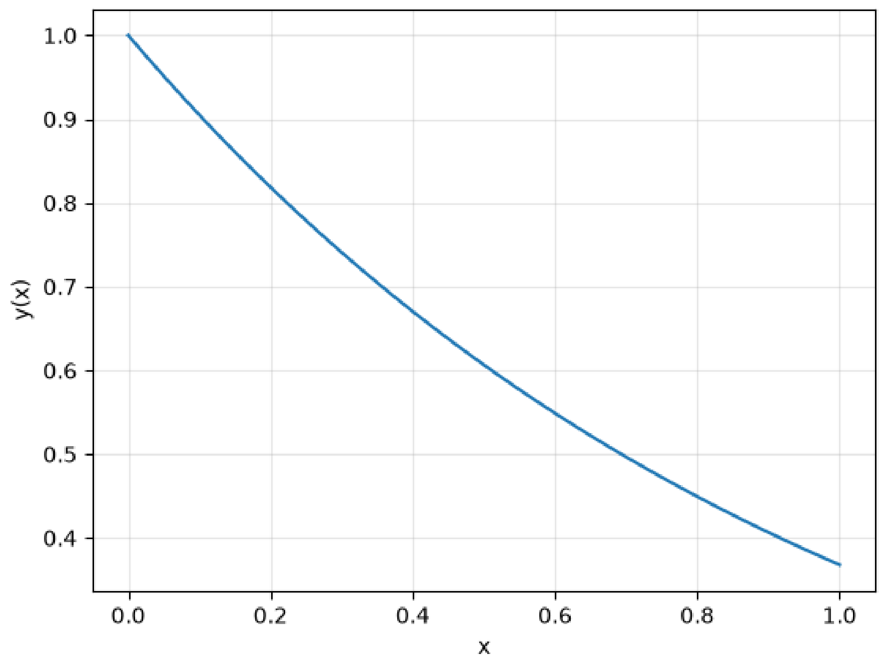


Figure 2: Solution on $[0, 1]$.

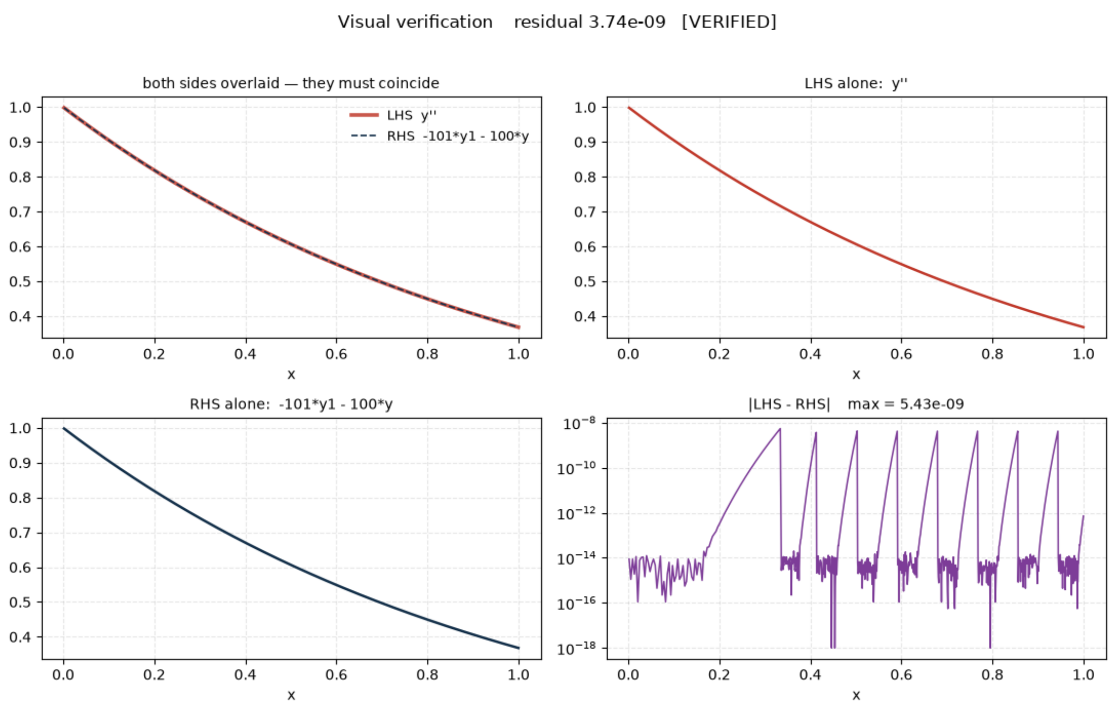


Figure 3: Verification on $[0, 1]$.

Part 3 — Pushed to $x = 10$

Then we made the interval ten times longer:

```
r = solve_ivp(order=2, f_str="-101*y1 - 100*y",
              x0=0.0, x1=10, ic={0: 1, 1: -1},
              iterations=20)
```

```
INFO: the interval [0.0, 10] exceeds a single series' reliable range;
      the solution was continued analytically over 111 segments
      (piecewise power series - still fully analytic on every piece).
INFO: run summary - series degree 20, iterations=20, 111 series segment(s).
INFO: verification by substitution - VERIFIED: satisfies the equation
      to 3.74e-09 on [0, 10].
```

Over a hundred analytic segments, assembled automatically, and the global verification residual is 3.74×10^{-9} — **the same figure as on** $[0, 1]$**.** Extending the interval tenfold did not degrade the certified accuracy; the segmentation absorbed the length, and the verification held.

Table 1: Three runs, one equation, zero tuning

Interval requested	Segments	Verified residual
none (open-ended)	1	10^{-6} relative, on $[0, 0.44398]$; beyond that, unaudited
$[0, 1]$	9	3.74×10^{-9}
$[0, 10]$	111	3.74×10^{-9}

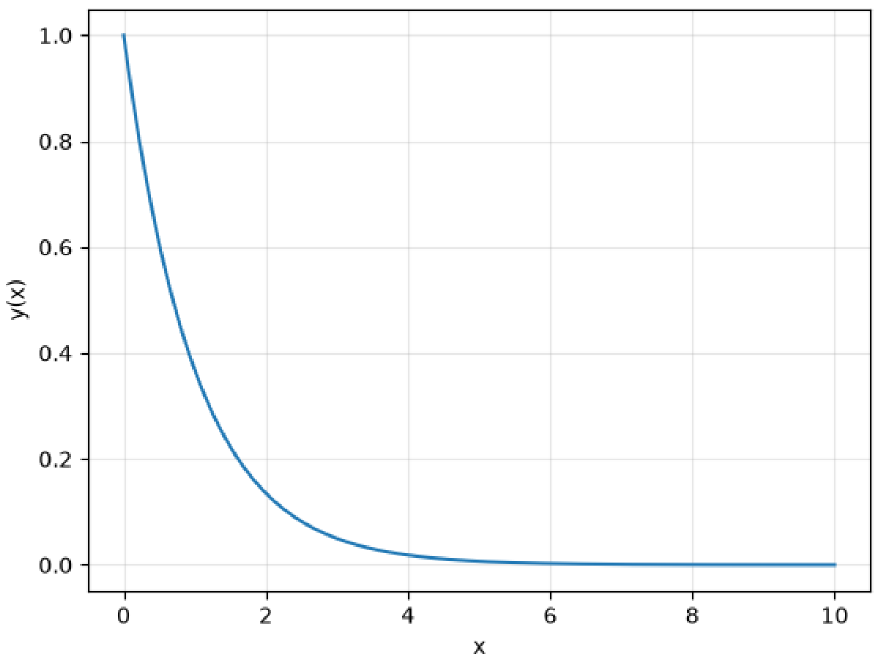


Figure 4: Solution on $[0, 10]$.

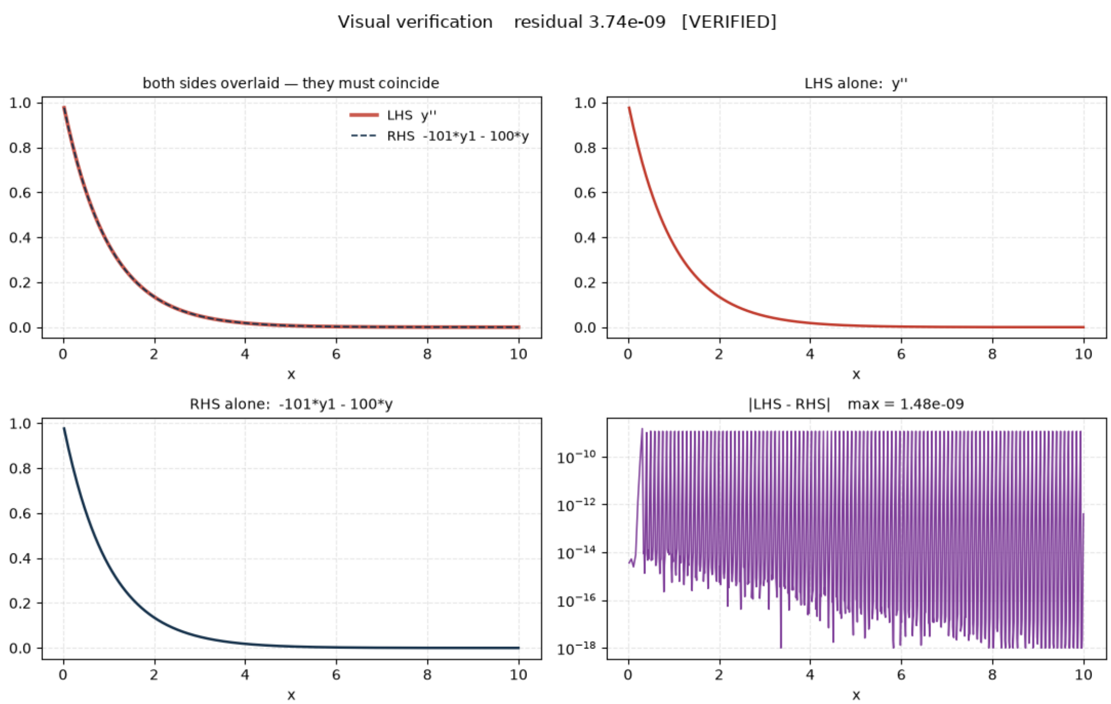


Figure 5: Verification on $[0, 10]$.

What this problem shows

- **The equation was entered as written.** Second order, as posed — no reduction to a first-order system, no solver-specific reformulation, and `iterations=20` was the only setting touched, identical across all three runs.
- **The guarantee has edges, and they are drawn for you.** The open-ended run states its reliable range, states the shorter verified interval, and labels everything beyond as unaudited — the distinction between *computed* and *checked*, made explicit.
- **Segmentation is automatic and disclosed.** Nine pieces for $[0, 1]$, over a hundred for $[0, 10]$, each an exact series sized by reliability, with the count reported.
- **Verification is global and by substitution** — the assembled solution put back into the equation across the whole interval, not merely matched at the seams — and it held at 3.74×10^{-9} on both the unit interval and the ten-fold one.

A note on scope

This is one stiff problem — a linear one, with a clean two-mode structure — and we present it as exactly that: an encouraging demonstration, not a claim that every stiff problem yields to zero preparation. Stiffness comes in degrees, and severely stiff problems remain hard for every method family. What this entry documents is the behaviour we consider non-negotiable regardless of difficulty: the solver states what it verified, marks what it did not, and never lets the second masquerade as the first.

The problem library

This entry is part of the SIRPY Problem Library — an ongoing, public catalog of mathematical systems solved and verified by SIRPY.

- **GitHub** — github.com/Mitra-Institute-of-Education-MIE/sirpy-problem-library
- **Web** — mitrainstituteofeducation.org/sirpy

Submit a problem

Have a differential or integral equation you believe is challenging, unusual, or simply beautiful? We would like to see it. We will run it through SIRPY and report honestly what happened — if it solves it, how; if it struggles, why; if it reaches a current limit, we will say so.

Contact: info@mitrainstituteofeducation.org · mitrainstituteofeducation.org/contact

Please only send problems you are free to share publicly.

SIRPY is in final validation prior to its first public release. Every line quoted above is verbatim from the solver's output; nothing is reproduced from memory.